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A NEW METHOD FOR THE PREDICTION OF AIRPLANE PERFORMANCE

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A NEW METHOD FOR THE PREDICTION OF AIRPLANE PERFORMANCE.

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Summary

A new method for the prediction of airplane performance in level and climbing flight, together with complete information regarding propeller speeds, is described in this report.

Developed from Bairstow's system and making use of American absolute coefficients, this method has advantages of simplicity and brevity.

An illustrative example is appended.

Introduction

The method of performance prediction which is described in this paper is a simplification and revision of Bairstow's method (Reference 1), which has been generally considered as highly accurate and dependable but also somewhat laborious and complicated. The new method is of advantage, particularly to the American aeronautical engineer, because it is formulated in terms of the American, rather than the British, system of absolute coefficients; it yields results which are as accurate and complete as those of the original method; it requires less labor,

and does not involve the use of the unfamiliar functions V/nP, $B.HP./N^3$, and $\frac{1}{4}\, k_T\, (V/nP)^{-2}$.

The new system, like all others, is merely a method of determining the conditions of equilibrium in steady flight. Although it is usual in performance prediction to consider the required and available power as variables dependent upon the air speed, the dependent variable might equally well be thrust, torque, or propeller speed. The use of a fictitious quantity, the product of the square root of the relative air density and the propeller speed, as the dependent variable is the distinguishing characteristic of the method described below. This product $\sigma^{1/2} N$, will be called the "indicated R.P.M." for lack of a better name and because of its apparent analogy to the indicated air speed $\sigma^{1/2} V$.

The advantages of this artifice are twofold. First, the determination of propeller (or engine) speed for any condition of steady flight requires no additional computation as its value is automatically fixed by the air speed and rate of climb (if any). Second, a single curve may be used to represent the airplane requirement at all altitudes, as the quantity "indicated R.P.M. required for level flight" has the unusual characteristic of depending only upon angle of attack, or indicated air speed.

Description of the Method

The required data are:

1. Weight, wing area and aerodynamic characteristics of the airplane (C_L vs. C_D).
2. Diameter and aerodynamic characteristics of the propeller (C_{T_2}, C_{P_1} and η vs. V/nD).
3. Sea level speed-power characteristics of the engine.
4. Relation between altitude and engine power at constant speed.

The reader's attention is invited to the forms of the propeller coefficients used here. The usual coefficients are

$$C_{T_1} = \frac{T}{\rho\ n^2\ \underline{D}^4} \quad C_{P_1} = \frac{P}{\rho\ n^3\ \underline{D}^5} \quad \eta = \frac{C_T}{C_{P_1}} \times \frac{V}{n\ \underline{D}}$$

From these are derived the less common forms

$$C_{T_2} = C_{T_1}\left(\frac{V}{n\ \underline{D}}\right)^{-2} = \frac{T}{\rho\ V^2\ \underline{D}^2} \qquad C_{P_2} = C_{P_1}\left(\frac{V}{n\ \underline{D}}\right)^{-3} = \frac{P}{\rho\ V^3\ \underline{D}^2}$$

All these coefficients are, of course, functions of $V/n\underline{D}$. For reasons which will appear, the forms C_{T_2} and C_{P_1} are most convenient for use with this method. The following simplifying assumptions are made:

a. Thrust is always parallel to the flight path.

b. Lift is always equal to weight.

The first assumption is made in the absence of knowledge of the direction of the resultant force acting upon a propeller which has its axis inclined to the direction of flight; it is partially justified by the known negligible reduction of thrust which accompanies small angles of pitch.

The second assumption follows directly from the first for the case of level flight. As applied to climbing flight, however, it is an approximation which improves in accuracy as the angle of climb decreases. The result is that rates of climb are slightly underestimated for low altitudes but negligibly so near the ceiling. In the illustrative example, this approximation leads to an error in the initial rate of climb of between 3 and 4 per cent. (It should be noted that the angle of climb is unusually large, approximately 13.7°.) If greater accuracy is desired, a second computation of the initial rate, involving the angle of climb found in the first, will give the desired result.

The prediction of the maximum speeds in level flight is accomplished by locating the intersections of the curve of "indicated R.P.M. required for level flight" vs. indicated air speed with the curves of "indicated R.P.M. available at full throttle" vs. indicated air speed. The indicated values are then divided by appropriate values of relative air density to obtain true air speed and R.P.M.

The minimum speeds are, of course, those corresponding to the maximum lift coefficient except in the neighborhood of the ceiling where the available indicated R.P.M. is insufficient to permit level flight at maximum lift coefficient; under this condition the minimum velocity is found by locating an intersection as in the case of maximum speed. The procedure of determining the coordinate of the curves of required and available indicated R.P.M. vs. indicated air speed, which are also used for the calculation of climbing performance, will now be detailed. A table of symbols will be found in the Appendix.

The conditions of equilibrium in level flight, according to the assumptions mentioned above, are

$$W = L \qquad \text{and} \qquad T = D$$

The assumption of an indicated air speed fixes the lift coefficient as

$$C_L = \frac{2\ W}{(\rho^{1/2}\ V)^2\ \ S}$$

The corresponding drag coefficient may be found by reference to the polar curve. The thrust coefficient may now be calculated by use of equation (1)

$$C_T = \frac{C_D\ S}{2\ \underline{D}^2} \tag{1}$$

which is derived by equating T and D in the forms

$$T = C_T\ \rho\ V^2\ \underline{D}^2$$

$$D = C_D\ S\ \rho\ V^2/2$$

The value of $V/n\underline{D}$ which corresponds to this thrust coefficient may be found from the chart of propeller characteristics. If we now combine the indicated air speed $\sigma^{1/2}\,V$, the propeller function $V/n\underline{D}$, and the propeller diameter $\underline{D}$, as follows:

$$\frac{\sigma^{1/2}V}{V/n\underline{D} \times \underline{D}} = \sigma^{1/2}n$$

we obtain the quantity $\sigma^{1/2}\,n$ which may be called the required "indicated r.p.s." and from which we may find $\sigma^{1/2}\,N$, the "indicated R.P.M." required for level flight. When V is in M.P.H., the value of $\sigma^{1/2}\,N$ is given by the equation

$$\sigma^{1/2}N = \frac{\sigma^{1/2}\,V_{\underline{MPH}} \times 88}{V/n\underline{D} \times \underline{D}} \qquad (2)$$

These calculations are to be duplicated for several indicated air speeds. The resulting curve of $\sigma^{1/2}\,N$ vs. $\sigma^{1/2}\,V$ is unique as it represents the airplane requirements for all altitudes; its independence of air density is demonstrated in the Appendix.

The values of <u>available</u> indicated R.P.M. (as function of indicated air speed) are given by a family of curves; each curve corresponds to a particular altitude. The determination of the coordinates of a point of one of these curves is based upon the following principle: The values of engine power (full throttle) and indicated R.P.M. are fixed by the assumption of an altitude and an engine speed. The power coefficient can then be calcu-

lated and the corresponding V/nD found by reference to the propeller characteristics chart. The indicated air speed at which the power supplied to and that absorbed by the propeller, at the known value of $\sigma^{1/2} N$ can now be computed since $\sigma^{1/2} N$, V/nD and D are known. This process has to be used only in the case of the sea level curve; the computations for other altitudes require less labor.

The details of the calculations for the sea-level condition are as follows: Having assumed a value of N, the power is found directly from the engine power curve. The power coefficient is then calculated according to the equation

$$C_P = \frac{P}{\rho\ n^3\ D^5} \qquad (3)$$

The corresponding value of V/nD is then determined by reference to the chart of propeller characteristics. Knowing V/nD and $\sigma^{1/2} N$, the latter of which at sea level, is equal to N, the indicated air speed (in M.P.H.) is

$$\sigma^{1/2} V_{MPH} = \sigma^{1/2} N \times V/nD \times \frac{1.466\ D}{60} \qquad (4)$$

A few repetitions of this calculation furnish the data from which the curve of available $\sigma^{1/2} N$ vs. $\sigma^{1/2} V$ for sea level may be plotted.

The work of calculating the coordinates of the curves for other altitudes is somewhat shortened by combining the calcula-

tions of engine power and power coefficient. If we assume that the engine power at any altitude is given by the equation

$$P_z = R_z \times P_o \tag{5}$$

wherein the subscripts o and z refer to sea level and any other altitude, respectively, and R_z, which is a function of the air temperature and pressure, is independent of the engine speed, the power coefficient for any altitude may be calculated by use of the equation

$$C_{P_z} = C_{P_o} \times \frac{R_z}{\sigma_z} \tag{6}$$

since $C_{P_z} = \dfrac{P_z}{\rho_z\, n^3\, \underline{D}^5} = \dfrac{P_o\ R_z}{\rho_o\ \sigma_z\, n^3\, \underline{D}^5} = \dfrac{P_o}{\rho_o\, n^3\, \underline{D}^5} \times \dfrac{R_z}{\sigma_z}$

and $C_{P_o} = \dfrac{P_o}{\rho_o\, n^3\, \underline{D}^5}$.

For the higher altitudes, then, one assumes a value of N and computes $\sigma^{1/2} N$. The value of C_{P_z} may be obtained directly from the sea level power coefficient (C_{P_o}) for the same value of N by the application of (6). The values of $V/n\underline{D}$ and $\sigma^{1/2} V$ are then obtained as outlined above for the sea-level condition.

Performance in climbing flight is also derived from this same set of curves by further computation. Bearing in mind the assumptions made at the outset, we may state the conditions of equilibrium in climb as follows:

$$W = L$$

$$T = D + W \frac{V_c}{V}$$

If the latter expression be put in coefficient form and W replaced by L, it becomes

$$C_T\ \rho\ V^2\ \underline{D}^2 = C_D\ \rho\ S\ \frac{V^2}{2} + C_L\ \rho\ S\ \frac{V^2}{2} \times \frac{V_c}{V}$$

Dividing through by $\frac{\rho\ SV^2}{2}$, we have

$$\frac{2\ C_T\ \underline{D}^2}{S} = C_D + C_L\ \frac{V_c}{V}$$

whence

$$\frac{V_c}{V} = \frac{\frac{2\ C_T\ \underline{D}^2}{S} - C_D}{C_L} \qquad (7)$$

The assumption of an indicated air speed fixes C_L and C_D. By reference to the curves (see Figure 4, for example), the available $\sigma^{1/2}\ N$ may be found and the value of $V/n\underline{D}$ computed according to the equation

$$V/n\underline{D} = \frac{\sigma^{1/2}\ V_{(MPH)}}{\sigma^{1/2} N} \times \frac{88}{\underline{D}} \qquad (8)$$

Having $V/n\underline{D}$, C_T can be found by use of the chart of propeller characteristics. Thus all of the quantities in the right-hand member of equation (7) are known and the ratio V_c/V is fixed. The rate of climb may now be calculated since

$$V_c\ (\text{ft./min.}) = \frac{V_c}{V} \times \frac{\sigma^{1/2} V_{(MPH)}}{\sigma^{1/2}} \times 88 \qquad (9)$$

The computation of the rates of climb at three or four indicated air speeds at each altitude is usually sufficient to establish the maximum rate of climb at that height. The plotted results of such computations for an illustrative example are shown as Figure 5. Having these curves, the variation with altitude of the indicated air speed for best climb may be determined by passing a curve through the maxima of the rate curves. This curve, when extrapolated to zero rate of climb, fixes the single value of indicated air speed at which level flight is possible at ceiling.

It is usually desirable to know the engine speed corresponding to maximum rate of climb at each altitude. Point values for the altitudes under consideration may now be determined by finding from the curves the values of available indicated R.P.M. corresponding to the indicated air speeds for best climb, and dividing them by $\sigma^{1/2}$.

This explanation is followed by an illustrative example. The reader's attention is invited, particularly, to the brevity of the required computations and curves in view of the completeness of the final results.

Illustrative Example

The performance of an airplane, which weighs 2075 pounds and has 284.5 square feet of wing area, is to be predicted. In Table I there are tabulated (a) the aerodynamic characteristics of the airplane; (b) the aerodynamic characteristics of the propeller (diameter 7.5 feet) and (c) the speed-power characteristics of the engine; these data are presented graphically as Figures 1, 2 and 3. The engine power at constant speed is assumed to vary directly with the atmospheric pressure and inversely with the square root of the absolute temperature, i.e., $R_z = \frac{p_z}{\sqrt{T_z}}$. (It is emphasized that this altitude power relation is entirely arbitrary and has no bearing upon the validity of the method of prediction as a whole. The essential qualification of such a relation is that it shall fit the particular power plant used. Special equations must be used, of course, when the engine is supercharged or overdimensioned.) The performance is to be calculated at the altitudes listed in Table II wherein there will be found, also, the characteristics of the standard atmosphere at those heights.

In Table III are the calculations leading to the curve or required $\sigma^{1/2} N$ vs. $\sigma^{1/2} V$ (curve AB) of Figure 4. Below the tabulated data will be found the equations used; these have been simplified by the substitution of the constants for this particular case.

The data from which the curves of available $\sigma^{1/2}N$ vs. $\sigma^{1/2}V$ have been plotted (Figure 4) will be found in Table IV along with the specific equations used. Attention is called to the fact that power coefficients (C_P) have been calculated for values of N which, at sea level, lie outside the range of flight. This is done because the calculations for higher altitudes necessitate the computation of power coefficients corresponding to small values of N and, as explained above, the work is simplified by making use of the sea level power coefficients. It is also pointed out that in some cases two values of V/nD are found to correspond to the same power coefficient. The reason for this is apparent in the shape of the power coefficient curve. The physical interpretation is that, with the engine at full throttle, there are two air speeds at which the same propeller speed may be expected. This feature is graphically represented by the appearance of a minimum value in each curve of available $\sigma^{1/2}N$ vs. $\sigma^{1/2}V$.

The performance characteristics in level flight, as derived from the curves of Figure 4, are summarized in Table V.

In Table VI are tabulated the computations leading to the determination of the rates of climb at various air speeds and altitudes. The result of these calculations are illustrated by Figure 5. More data than were necessary for the establishment of the maxima have been computed and plotted to illustrate the shape of the rate of climb curves over the entire speed range.

The summary of the climbing performance is given in Table VII. The values of V_c max. and the corresponding indicated air speeds are defined by the intersections of the curve CD with the rate of climb curves of Figure 5.

The collected results of the performance prediction are presented in graphical form as Figure 6. Curves of rate of climb, maximum true air speed, indicated air speed of best climb, R.P.M. in level flight, and R.P.M. in climb, all plotted against altitude, will be found there. The ceiling is indicated by the rate of climb curve; the corresponding speed and R.P.M. have been calculated for this altitude in order that the curves could be properly faired above the highest altitude for which routine computations have been made.

The only curve which seems to require any comment is that of R.P.M. in level flight at minimum speed. The sharp break in the neighborhood of 22,000 feet is the result of there being available, above this altitude, insufficient power to maintain level flight at air speeds corresponding to the maximum lift coefficient. At higher altitudes the minimum speed of level flight corresponds to lower lift coefficients and, consequently, to reduced values of required power and of $\sigma^{1/2}N$.

A p p e n d i x

The representation of the level flight requirements of an airplane at all altitudes by a single curve is not a new conception, but, as it has not had much use in this country, it will be explained in detail. It is to be demonstrated that the indicated R.P.M. $(\sigma^{1/2} N)$ required to maintain an airplane in level **flight** at a given angle of attack is independent of altitude. Two proofs are given. The first is a direct analytical proof, while the second is presented in an effort to give a more complete physical picture of the inter-relation of the quantities involved.

The drag of a given airplane at a particular angle of attack in level flight is fixed by the ratio L/D and is independent of air density. As the thrust is assumed to equilibrate the drag, the thrust, also, is not affected by variations of density.

The thrust of a given propeller is a function of the ratio $V/n\underline{D}$ and of the dynamic pressure $\rho V^2/2$, or q, i.e.,

$$T = f(V/n\underline{D},\ q) \qquad \text{(a)}$$

or, in explicit form

$$T = C_T\ \rho\ V^2\ \underline{D}^2$$

wherein C_T is $f(V/n\underline{D})$.

The stipulation of level flight at a particular angle of attack fixes q. Equation (a) then consists of $f(V/n\underline{D})$ and two constants T and q; consequently, $V/n\underline{D}$ and V/n are constants. It is thus shown that n must vary directly with V. As $\sigma^{1/2} V$, the indicated air speed, is constant, $\sigma^{1/2} N$ must also be constant.

While the foregoing proof is complete in itself, the treatment which follows may be of some assistance in clarifying the physical background.

Consider an airplane which is assumed to be in level flight at sea level at some particular speed. The angle of attack and the R.P.M. of the propeller are, therefore, definitely fixed. Now let us assume that the airplane ascends to some considerable altitude and is flown horizontally at the same angle of attack as at sea level. The reduction of air density necessitates the increase of the true air speed to such a value that the dynamic pressure at the high altitude shall be the same as it was at sea level, i.e., $\rho \ V^2/2$ or $\sigma^{1/2} V =$ constant. The drag of the airplane remains unchanged inasmuch as the L/D is a function of angle of attack only. Since the thrust is assumed to equilibrate the drag, the former must also remain constant so long as the angle of attack is not changed.

How may this thrust be produced – at an increased true air speed through air of reduced density? Let us deal with a single element of the propeller blade and see how its thrust

may be maintained at the same value as that for the lower altitude. The force upon this elementary airfoil is dependent upon two variables: the angle of attack and the dynamic pressure corresponding to the speed of the blade element. If these quantities have the same values as in the previous condition, the force on the blade element and, consequently, the thrust of the whole propeller will remain unchanged. The angle of attack is fixed by the ratio V/nD. Let us find the results of reproducing the sea level V/nD at the higher altitude. As D is fixed, N must vary directly with V. Therefore ρN^2 or $\sigma^{1/2} N$ must remain constant. We must now find the dynamic pressure. Since V/nD is unchanged the path of the blade element is geometrically similar to that which it described at the lower altitude. Therefore, the true velocities of the blade element at the two altitudes and the corresponding dynamic pressures are proportional, respectively, to the velocities of flight and their corresponding dynamic pressures. As we know, the last mentioned quantities to be equal, the maintenance of the same V/nD with the airplane in level flight at a given angle of attack produces a constant thrust at all altitudes. This was the condition which was to be satisfied. Thus it has been shown that with a constant V/nD the indicated R.P.M. $\sigma^{1/2} N$, remains constant under these conditions.

W = weight, lb.

L = lift, lb.

D = drag, lb.

T = thrust, lb.

$\underline{D}$ = propeller diameter, ft.

S = wing area, sq.ft.

P = power, ft.lb./sec.

B.HP. = brake horsepower

V = flight velocity, ft./sec.

V_c = rate of climb, ft./sec.

ρ = air density, slugs/cu.ft.

σ = relative air density (ρ/ρ_o)

p = air pressure (in. Hg)

T = air temperature (degrees Fahrenheit, abs.)

q = dynamic pressure $(\rho V^2/2)$, lb./sq.ft.

n = propeller speed, r.p.s.

N = propeller speed, R.P.M.

C_L = lift coefficient (L/qS)

C_D = drag coefficient (D/qS)

C_T = thrust coefficient $(T/\rho V^2 \underline{D}^2)$

C_P = power coefficient $(P/\rho n^3 \underline{D}^5)$

$\sigma^{1/2} V$ = indicated air speed, ft./sec.

$\sigma^{1/2} N$ = "indicated R.P.M."

R = power ratio (P/P_o)

TABLE I.

Airplane, Engine and Propeller Characteristics

(a) Airplane

C_L	C_D
0.0	0.0470
.2	.0480
.4	.0555
.6	.0690
.8	.0890
1.0	.1135
1.2	.1525
1.3	.1900
1.335	.2300
1.322	.2500

(c) Engine

R.P.M.	B.HP.
1500	189.7
1600	201.8
1700	213.7
1800	225.0
1900	235.3
2000	244.9

(b) Propeller

V/nD	C_P	η	C_T
0.3	0.0870	0.487	1.570
.35	.0877	.544	1.113
.4	.0880	.594	.817
.5	.0872	.679	.474
.6	.0845	.744	.291
.7	.0802	.788	.184
.8	.0733	.809	.116
.9	.0629	.805	.0695
1.0	.0498	.752	.0375

$$C_P = \frac{P}{\rho\ n^3\ D^5}$$

$$C_T = \frac{T}{\rho\ V^2\ D^2} = \frac{\eta\ C_P}{(V/nD)^3}$$

TABLE II.

Altitude Relations Based on Standard Atmosphere
(Reference 2)

Altitude (ft.)	Pressure ratio $\left(\frac{p}{p_o}\right)$	Temp. ratio $\left(\frac{T}{T_o}\right)$	Density ratio $\left(\frac{\rho}{\rho_o} = \sigma\right)$	Power ratio (R)	$(\sigma^{\frac{1}{2}})$	$\frac{R}{\sigma}$
0	1.000	1.000	1.000	1.000	1.000	1.0000
5000	.832	.965	.862	.847	.928	.9825
10000	.687	.931	.738	.712	.859	.9650
15000	.564	.897	.629	.596	.793	.9475
20000	.459	.862	.533	.494	.730	.9260
25000	.371	.828	.448	.408	.669	.9110
30000	.297	.794	.374	.334	.612	.8905

$$R = \frac{P}{P_o} = \frac{p}{p_o}\sqrt{\frac{T_o}{T}}$$

TABLE III.

Airplane Requirements

(Level Flight)

$\sigma^{\frac{1}{2}}$ V (M.P.H.)	C_L	C_D	C_T	V/n$\underline{D}$	$\sigma^{\frac{1}{2}}$ N
46.6	1.322	.2500	.6320	.443	1234
46.3	1.335	.2300	.5817	.459	1183
48.0	1.242	.1650	.4170	.525	1073
50	1.145	.1389	.3511	.560	1047
55	.946	.1058	.2673	.617	1046
60	.795	.0875	.2212	.659	1068
70	.584	.0675	.1707	.717	1145
80	.447	.0579	.1465	.750	1252
90	.353	.0528	.1335	.772	1367
100	.286	.0500	.1265	.781	1501
110	.236	.0488	.1234	.787	1640
120	.199	.0480	.1214	.790	1782
130	.169	.0474	.1199	.794	1921

$$C_L = \frac{2\ W}{\sigma\ \rho_o\ V^2\ S} = \frac{2 \times 2075}{.00237 \times 1.463^2 \times 284.5} \times \frac{1}{\sigma\ V^2} = \frac{2861}{\sigma^{\frac{1}{2}}\ V^2}$$

$$C_T = \frac{T}{\rho\ V^2\ \underline{D}^2} = \frac{D}{\rho\ V^2\ \underline{D}^2} = \frac{C_D\ \rho\ S\ V^2}{2\ \rho\ V^2\ \underline{D}^2} = \frac{C_D\ S}{2\ \underline{D}^2} = \frac{284.5}{2 \times 7.5^2}\ C_D$$

$$= 2.529 C_D$$

$$\sigma^{\frac{1}{2}}\ N = \frac{\sigma^{\frac{1}{2}}\ V \times 60}{V/n\underline{D} \times \underline{D}} = \frac{60 \times 1.466}{7.5} \times \frac{\sigma^{\frac{1}{2}}\ V}{V/n\underline{D}} = 11.733\ \frac{\sigma^{\frac{1}{2}}\ V}{V/n\underline{D}}$$

TABLE IV.

Engine - Propeller Output, Full Throttle

N (R.P.M.)	$B.HP_o$	$\sigma^{\frac{1}{2}}$ N (R.P.M.)	C_p	V/nD		$\sigma^{\frac{1}{2}}$ V (M.P.H.)	
Sea level							
1950	240.0	(Same as N)	0.0684	0.852		141.5	
1900	235.3		.0725	.811		131.1	
1850	230.2		.0768	.755		118.9	
1800	225.0		.0815	.675		103.5	
1750	219.4		.0866	.529	(.280)	78.9	(41.8)
1740	218.3		.0875	.475	(.334)	70.4	(49.5)
1725	216.5		.0892				
1710	214.9		.0909				
1700	213.7		.0919				
1675	210.9		.0948				
5,000 ft.							
1900	(As above)	1764	0.0712	0.823		123.7	
1850		1717	.0755	.773		113.1	
1800		1671	.0801	.702		100.0	
1750		1625	.0851	.583		80.8	
1725		1601	.0876	.467	(.346)	63.7	(47.2)
10,000 ft.							
1850	(As above)	1590	0.0741	0.791		107.2	
1800		1547	.0786	.728		96.0	
1750		1504	.0836	.626		80.2	
1725		1481	.0861	.546		68.9	
1710		1470	.0876	.465	(.340)	58.2	(42.6)
15,000 ft.							
1850	(As above)	1468	0.0778	0.806		100.8	
1800		1428	.0772	.749		91.8	
1750		1388	.0821	.662		79.9	
1725		1367	.0845	.600		70.0	
1700		1349	.0871	.503	(.305)	59.0	(35.1)

TABLE IV (Cont.)

Engine - Propeller Output, Full Throttle

N (R.P.M.)	B.HP$_o$	$\sigma^{\frac{1}{2}}$ N (R.P.M.)	C_P	V/nD		$\sigma^{\frac{1}{2}}$ V (M.P.H.
20,000 ft.						
1800	(As above)	1314	0.0754	0.775		86.8
1750		1278	.0802	.700		76.2
1725		1260	.0826	.650		69.8
1700		1241	.0851	.582		61.6
1675		1223	.0878	.440	(.365)	45.9 (38.0)
25,000 ft.						
1800	(As above)	1205	0.0742	0.789		81.0
1750		1171	.0789	.722		72.9
1725		1154	.0812	.682		67.1
1700		1137	.0837	.624		60.5
1675		1121	.0864	.535		51.2
30,000 ft.						
1750	(As above)	1070	0.0771	0.750		68.4
1725		1055	.0795	.715		64.3
1700		1040	.0818	.667		59.1
1675		1024	.0844	.604		52.7

$$C_P = \frac{P}{\rho\ n^3\ \underline{D}^5} = \frac{B.HP_o \times R \times 550 \times 60^{-3}}{\rho_o\ \sigma\ N^3\ \underline{D}^5} = 2,112,300\ \frac{B.HP.}{N^3} \times \frac{R}{\sigma} =$$

$$= C_{P_o} \frac{R}{\sigma}$$

$$\sigma^{\frac{1}{2}}\ V\ (M.P.H.) = \sigma^{\frac{1}{2}}\ N \times V/n\underline{D} \times \frac{\underline{D}}{60} \times \frac{1}{1.466} = 0.0852\ \sigma^{\frac{1}{2}}\ N \times V/n\underline{D}$$

TABLE V.

Performance in Level Flight

Altitude ft.	$\sigma^{\frac{1}{2}} V_{max}$ (M.P.H.)	$\sigma^{\frac{1}{2}}$ N (R.P.M.)	V_{max} (M.P.H.)	N (R.P.M.)
0	127.3	1883	127.3	1883
5000	116.8	1734	125.9	1869
10000	106.0	1584	123.5	1844
15000	96.1	1448	121.1	1825
20000	85.2	1309	116.6	1793
25000	73.0	1175	109.0	1756
29100	55.5	1048	89.2	1682
	$\sigma^{\frac{1}{2}} V_{min}$		V_{min}	
20000	46.6	1183	63.8	1620
25000	46.8	1118	70.0	1670

TABLE VI.

Performance in Climb

$\sigma^{\frac{1}{2}} V$ (M.P.H.)	$\sigma^{\frac{1}{2}} N$ (R.P.M.)	V/nD	C_T	C_L	C_D	$\left(\frac{2C_T D^2}{S}\right)$	$\left(\frac{2C_T D^2}{S} - C_D\right)$	$\frac{V_c}{V}$	V_c (ft./min.)
Sea level									
46.3	1743	0.312	1.453	1.335	0.230	0.574	0.344	0.257	1048
50	1739	.337	1.218	1.145	.139	.481	.342	.299	1316
60	1736	.405	.796	.795	.087	.314	.227	.285	1505
70	1740	.472	.549	.584	.067	.217	.150	.257	1584
80	1751	.536	.394	.447	.058	.156	.098	.219	1541
90	1768	.597	.295	.353	.053	.117	.064	.181	1434
5,000 ft.									
46.3	1602	0.339	1.188	1.335	0.230	0.469	0.239	0.177	778
50	1600	.367	.993	1.145	.139	.392	.253	.221	1048
60	1600	.440	.649	.795	.087	.257	.170	.214	1216
70	1608	.511	.448	.584	.067	.177	.110	.118	1250
80	1623	.578	.322	.447	.058	.127	.069	.154	1170
90	1644	.642	.239	.353	.053	.095	.042	.119	1015
10,000 ft.									
46.3	1468	0.370	0.974	1.335	0.230	0.385	0.155	0.116	552
50	1468	.400	.817	1.145	.139	.322	.183	.160	818
60	1472	.478	.531	.795	.087	.210	.123	.155	951
70	1483	.554	.362	.584	.067	.143	.076	.130	934
80	1501	.621	.263	.447	.058	.104	.046	.103	844
15,000 ft.									
46.3	1342	0.405	0.791	1.335	0.230	0.313	0.083	0.062	320
50	1342	.437	.660	1.145	.139	.261	.122	.107	591
60	1350	.521	.424	.795	.087	.168	.081	.102	678
70	1367	.600	.291	.584	.067	.115	.048	.082	639
80	1392	.674	.211	.447	.058	.083	.025	.056	496

TABLE VI (Cont.)

Performance in Climb

$\sigma^{\frac{1}{2}} V$ (M.P.H.)	$\sigma^{\frac{1}{2}} N$ (R.P.M.)	V/nD	C_T	C_L	C_D	$\left(\frac{2C_T D^2}{S}\right)$	$\left(\frac{2C_T D^2}{S} - C_D\right)$	$\frac{V_c}{V}$	V_c (ft./min.)
20,000 ft.									
46.3	1222	0.444	0.632	1.335	0.230	0.250	0.020	0.015	84
50	1223	.480	.527	1.145	.139	.208	.069	.060	363
60	1238	.569	.336	.795	.087	.133	.046	.058	419
70	1261	.651	.229	.584	.067	.091	.024	.041	347
25,000 ft.									
50	1119	0.524	0.418	1.145	0.139	0.165	0.026	0.0227	149
60	1137	.619	.266	.795	.087	.105	.018	.0226	178
70	1166	.704	.181	.584	.067	.072	.005	.0086	79

$$C_L = \frac{2861}{(\sigma^{\frac{1}{2}} V)^2}$$

$$\frac{V_c}{V} = \frac{\frac{2\ C_T\ D^2}{S} - C_D}{C_L}$$

$$V_c \text{ (ft./min.)} = \frac{V_c}{V} \times \frac{\sigma^{\frac{1}{2}} V_{(MPH)}}{\sigma^{\frac{1}{2}}} \times 88$$

TABLE VII.

Performance in Climb - (Summary)

Altitude (ft.)	V_c max (ft./min.)	$\sigma^{\frac{1}{2}}$ V (M.P.H.)	$\sigma^{\frac{1}{2}}$ N (R.P.M.)	N (R.P.M.)
0	1585	71.8	1742	1742
5000	1252	67.0	1605	1730
10000	957	63.5	1474	1716
15000	679	60.4	1350	1702
20000	420	58.0	1234	1690
25000	181	56.2	1129	1688

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1. Bairstow, Leonard : "Applied Aerodynamics," Chapter IX (1920).
2. Diehl, Walter S. : "Standard Atmosphere - Tables and Data." (1925)

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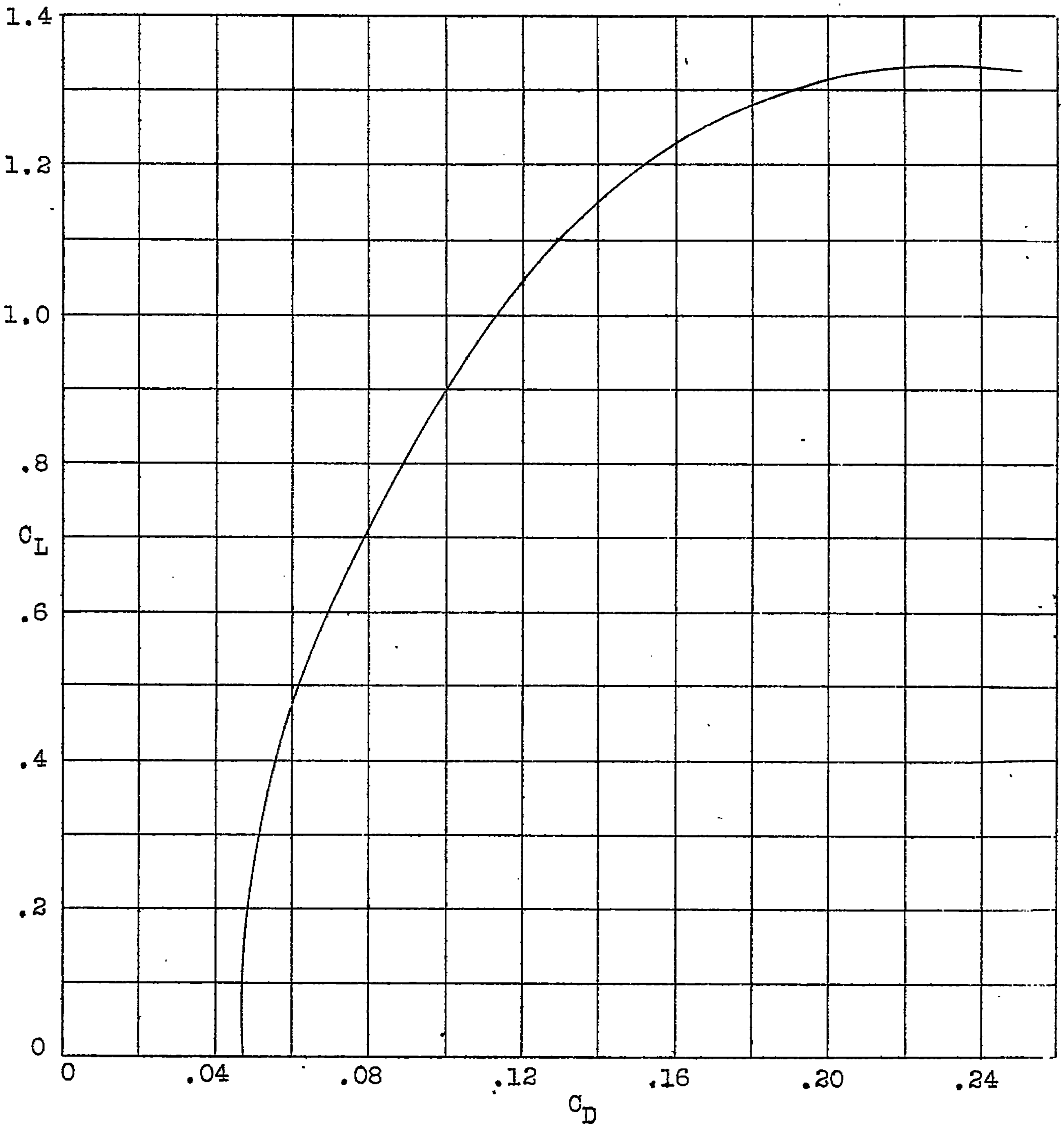


Fig.1

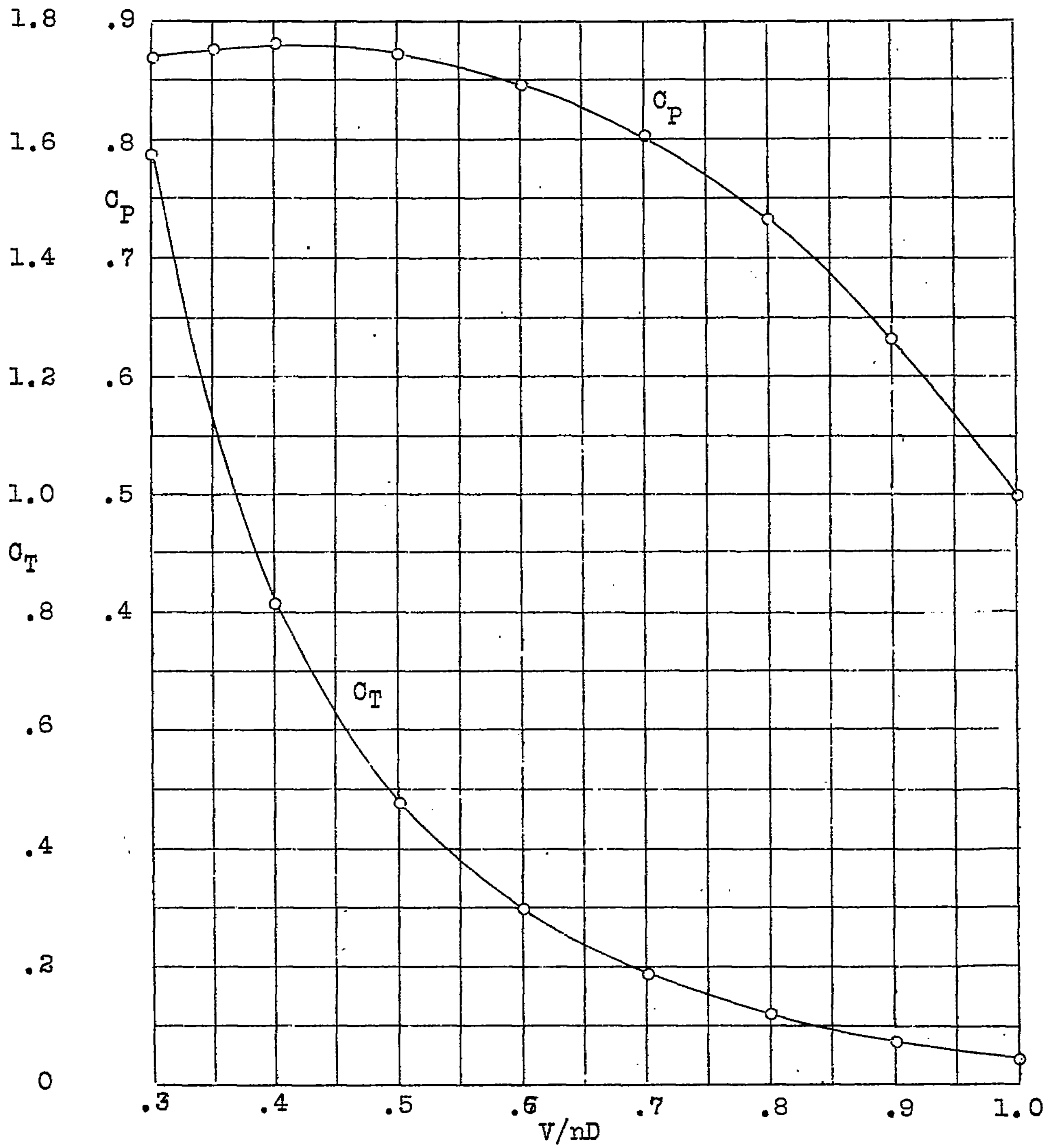


Fig.2

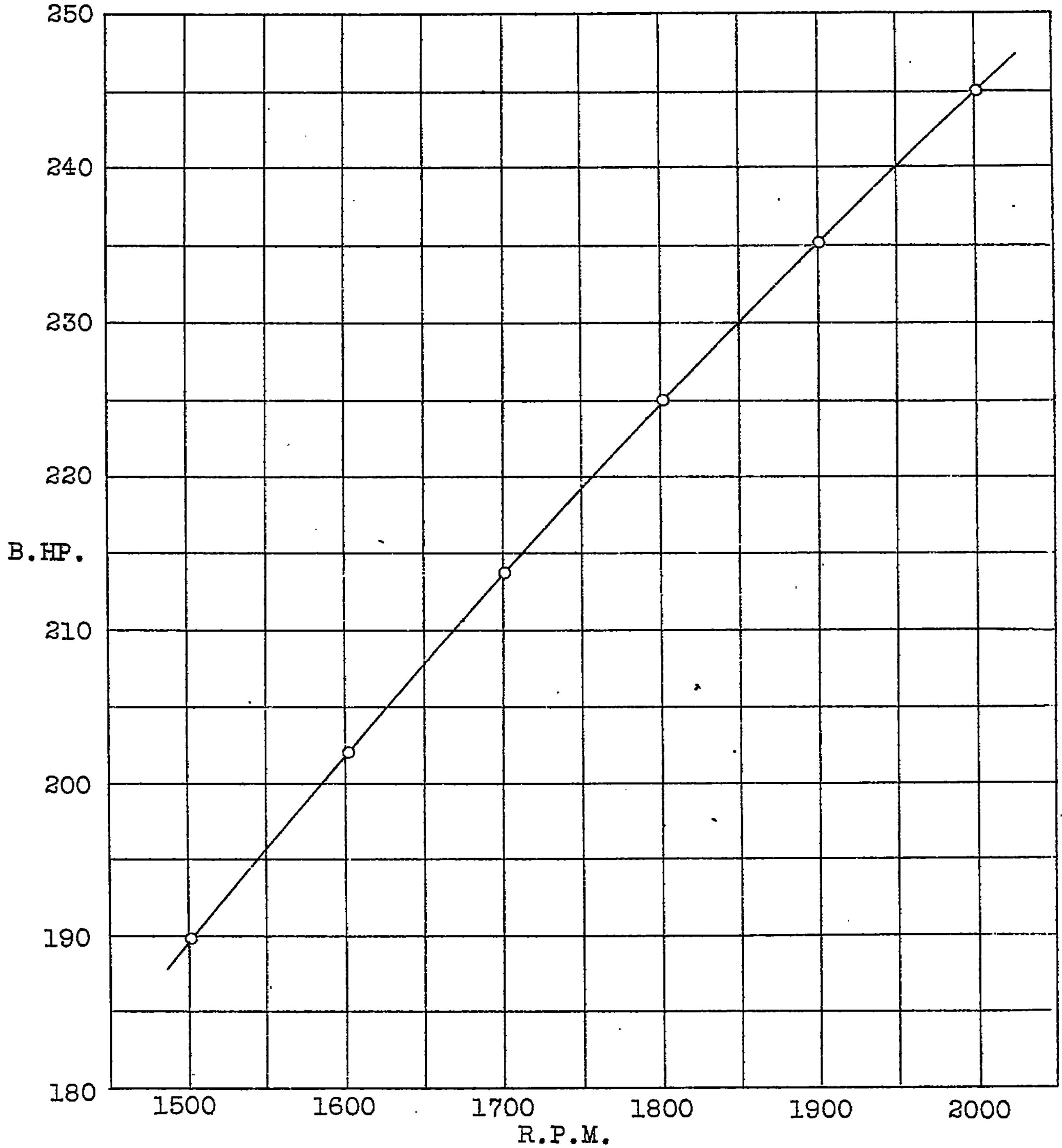
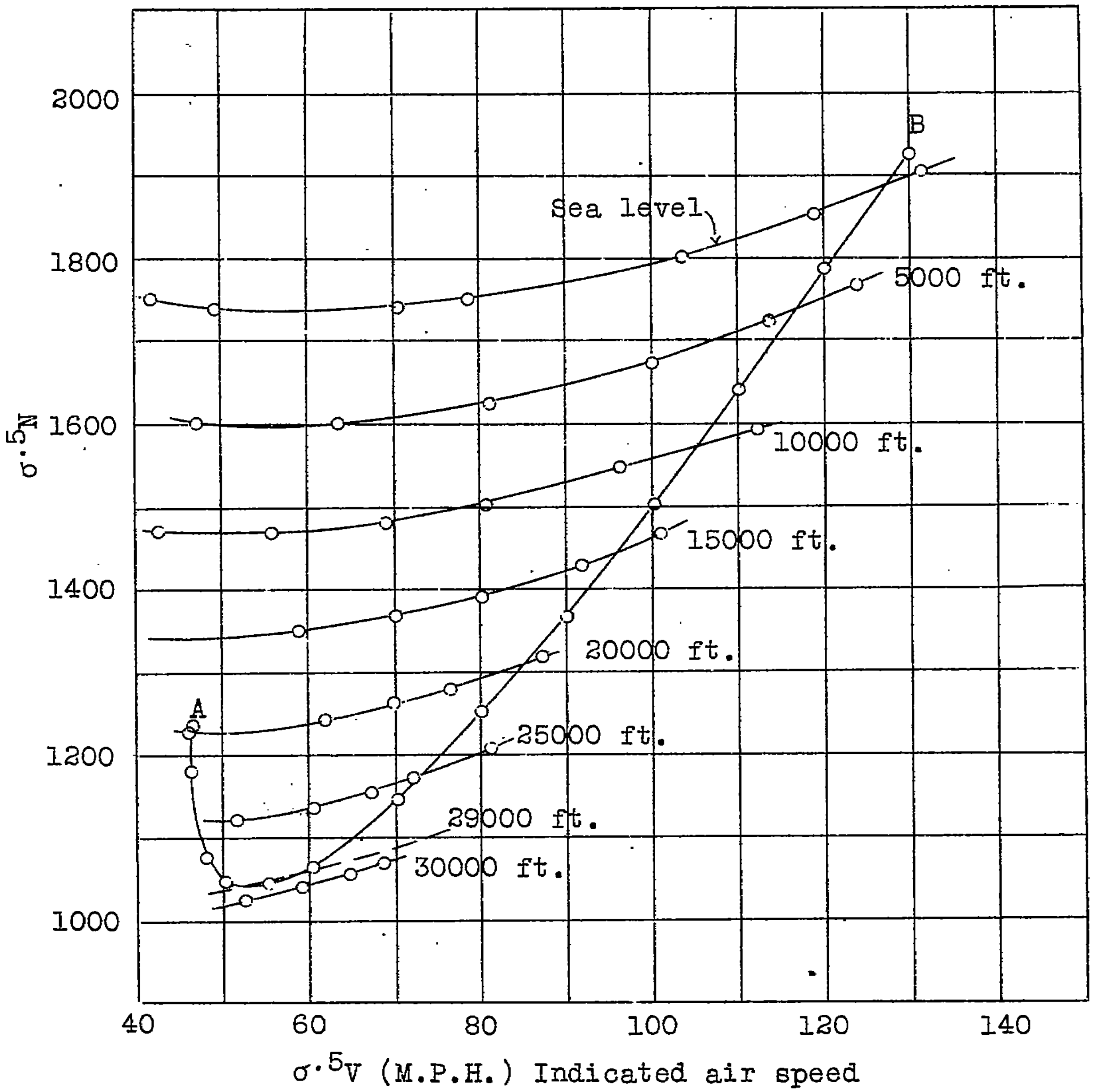
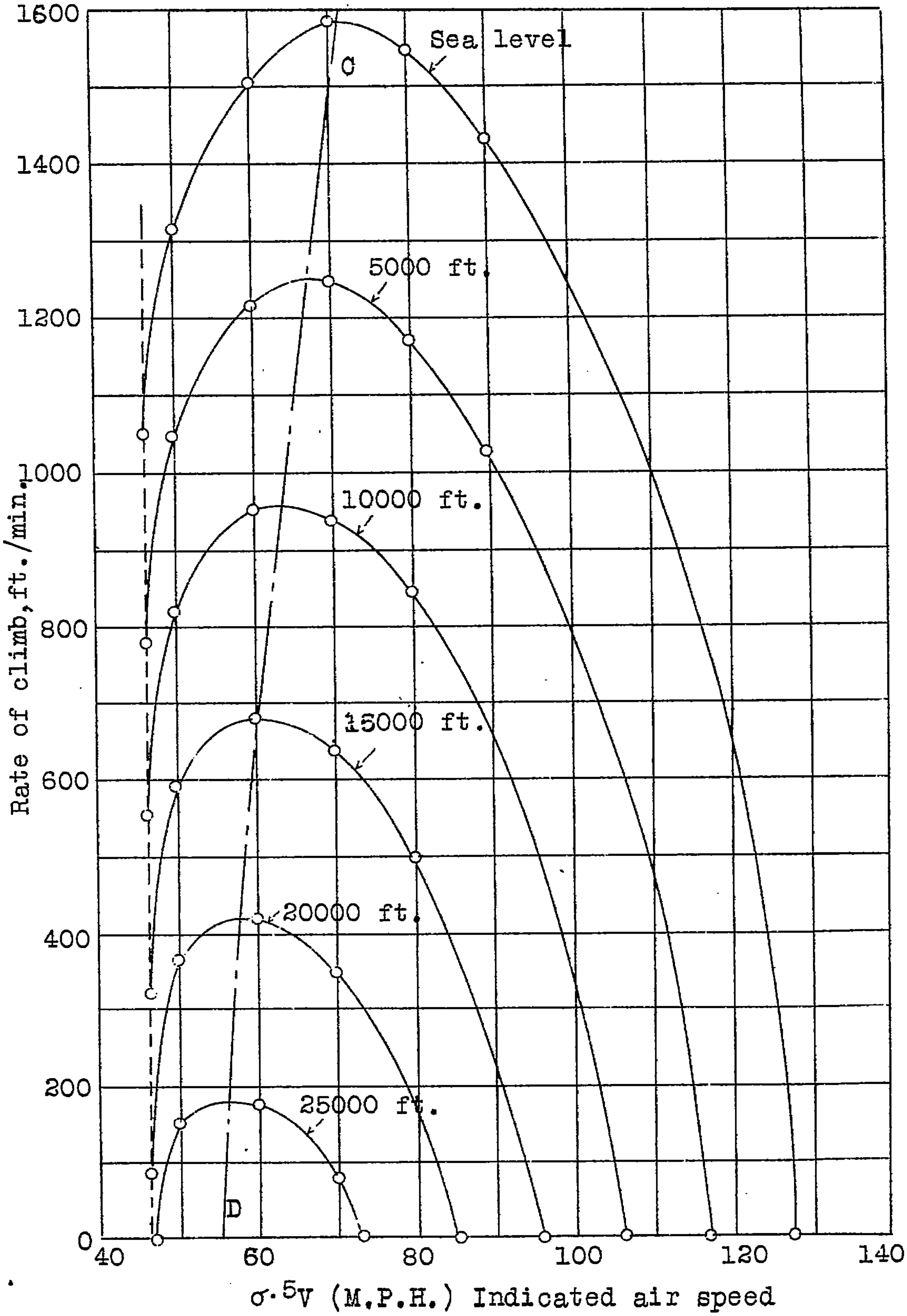

250
240
230
220
B.HP.
210
200
190
180
1500
1600
1700
1800
1900
2000
R.P.M.

Fig.3



2000
1800
1600
1400
1200
1000
$\sigma^{.5}N$
40
60
80
100
120
140
$\sigma^{.5}V$ (M.P.H.) Indicated air speed
A
B
Sea level
5000 ft.
10000 ft.
15000 ft.
20000 ft.
25000 ft.
29000 ft.
30000 ft.

Fig.4



1600
1400
1200
1000
800
600
400
200
0
Sea level
C
5000 ft.
10000 ft.
15000 ft.
20000 ft.
25000 ft.
D
Rate of climb, ft./min.
40
60
80
100
120
140
$\sigma^{.5}$V (M.P.H.) Indicated air speed

Fig.5

N.A.C.A. Technical Note No.302

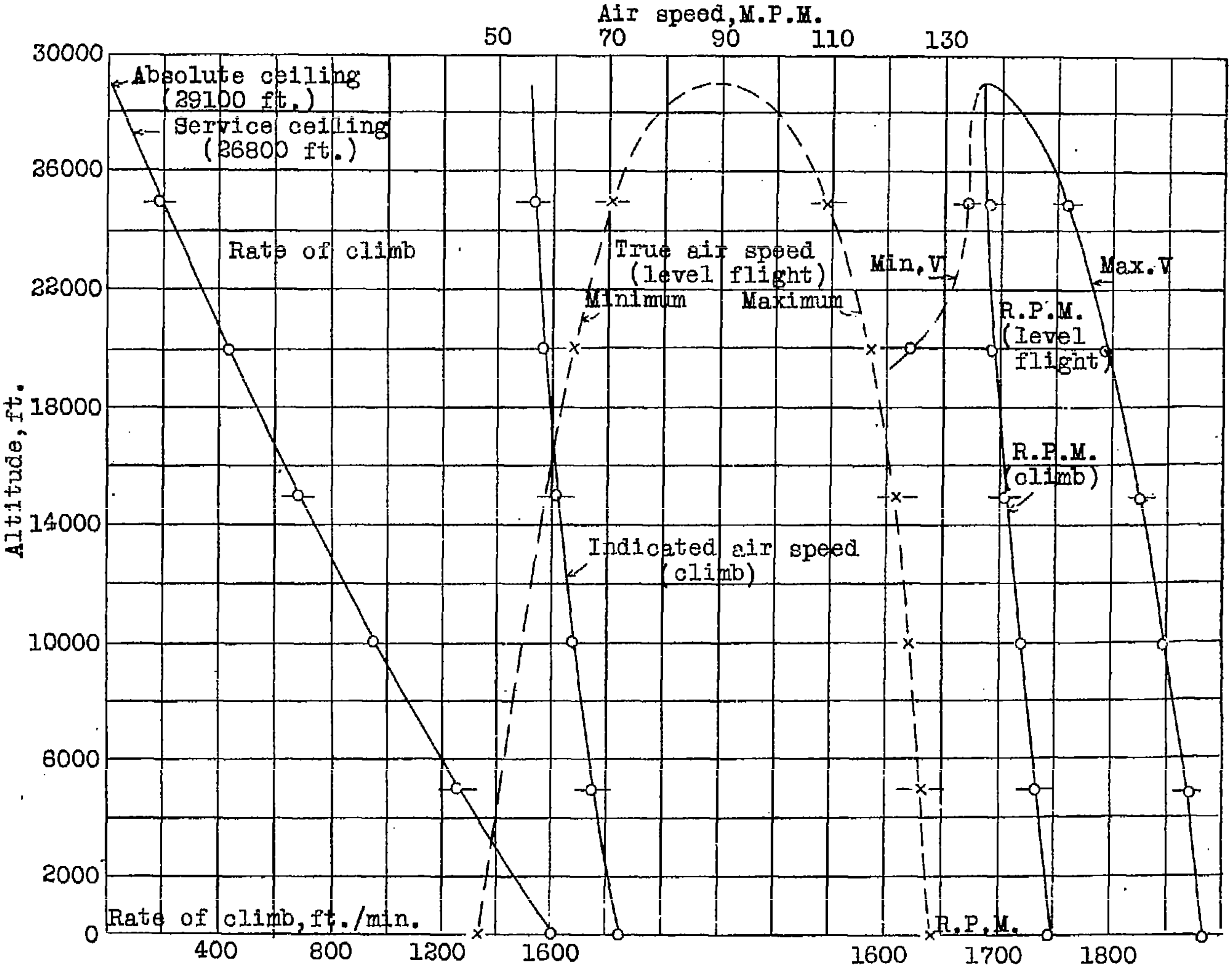


Fig.6